**Advanced Algorithms**

**Exercise for Lecture 15**

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| **Student Name** |  | **Student ID** |  |
| **Problem 1** |  | | |
| **Problem 2** |  | | |
| **Problem 3** |  | | |
| **Total Score** |  | | |
| **Notes** | Deadline: **2023-11-13 24:00**  Submission Format: ‘**Lecture15\_Name\_Student ID.docx**’, and please send to: **[chenlq1997@126.com](mailto:algorithms_23fall@163.com)**.  This assignment is meant to be an evaluation of your **individual** understanding coming into the course and should be completed **without collaboration** or outside help. | | |

**Problem 1.[30 points]**

There exists two couriers, they must deliver a number of parcels to a number of addresses. They will go to the express point first every day to get the parcels. And they will go back to the express point at the end of every day to report the work. This gives the following decision problem.

**Instance:** Set *V* of locations. For each pair of locations *v, w ∈ V* , there is a distance *d*(*v, w*) *∈* N, a starting location *s ∈ V* , and an integer *K*.

**Question:** Are there two cycles, that both start in *s*, such that every location in *V* is on at least one of the two cycles, and both cycles have length at most *K*?

Show that this problem is NP-complete.

**Solution:**

**Problem 2.[30 points]**

Please prove that clique problem is NPC. Given that 3-SAT problem is a NPC problem.

For a given undirected graph G=(V,E), the clique of it is V’ which is a subset of V. For any point pair u,v∈V’, the edge (u,v)∈E. And the clique problem is to find the max clique which contains most points of the graph G.

**Solution:**

**Problem 3.[40 points]**

Please prove that vertex cover problem is NPC. Given that clique problem is a NPC problem.

For a given undirected graph G=(V,E), the vertex cover of it is a V’ which is a subset of V. For any edge (u,v)∈E, the point u∈V’ or v∈V’(or both u,v∈V’). That is to say for every edge that belong to E, at least one point of the edge belongs to the vertex set V’, and then V’ is a vertex cover of G. The vertex cover problem is to find a min vertex cover which contains least points of the graph G.

We are given an undirected graph (*V, E*). A vertex cover is a subset *W* ⊆ *V* such that for each (*v, w*) *∈ E* we have *v ∈ W* or *w ∈ W*. We consider the following problem. Vertex Cover

**Instance:** Undirected graph *G* = (*V, E*), integer *K*.

**Question:** Does *G* have a vertex cover of at most *K* vertices?

3.1 Show that Vertex cover belongs to the class NP.

3.2 Proof that the Vertex Cover problem is NP-complete by a reduction from max clique problem.

**Solution:**